1. Propositions: Statements that have a truth value T or F

2. Operate on propositions

  Unary: \( \neg P \)
   Binary: \( P, q \)

   \( \neg P \), \( P \lor q \), \( P \land q \), \( P \oplus q \)

   \( P \rightarrow q \), \( P \iff q \)

3. Compare propositions.

   The propositions are "equal" logically equivalent.

   Two propositions are logically equivalent if and only if they have the same truth values.
A few common logical equivalences:

\[
\begin{align*}
\neg (\neg p) & \equiv p \\
\neg (p \land q) & \equiv \neg p \lor \neg q \\
\neg (p \lor q) & \equiv \neg p \land \neg q \\
\neg p \land \neg q & \equiv \neg (p \lor q) \\
\neg p \lor \neg q & \equiv \neg (p \land q) \\
\end{align*}
\]

De Morgan's laws:

\[
\begin{align*}
\neg (p \lor q) & \equiv \neg p \land \neg q \\
\neg (p \land q) & \equiv \neg p \lor \neg q \\
\end{align*}
\]

Distribution:

\[
\begin{align*}
p \lor (q \land r) & \equiv (p \lor q) \land (p \lor r) \\
p \land (q \lor r) & \equiv (p \land q) \lor (p \land r) \\
\end{align*}
\]
1) Quantifies

when I was writing the sentence

\[ x + 1 = 3 \]

This sentence is not a proposition because:

a) I do not know the domain of \( x \)

b) I need to decide if I want that sentence to be true for all values of \( x \) (in the domain) or for some values of \( x \)

For all \( x \) in the set of real numbers

\[ x + 1 = 3 \]

This is now a proposition.

Truth value: False
\[ P(x) : \quad x + 1 = 3 \]

**Predicate:**

\[ \forall x \in \mathbb{R}, \quad x + 1 = 3 \]

**Proposition:**

\[ \forall x \in \mathbb{R}, \quad x + 1 = 3 \]

This proposition is **false**.

**There exists:**

\[ \exists x \in \mathbb{R}, \quad x + 1 = 3 \]

This proposition is **true**.

**Validation a quantified predicate:**

\[ \forall x \in \mathbb{D}, \quad P(x) \]

- **True:**
  - \( P(x) \) is true for all \( x \) in \( \mathbb{D} \)

- **False:**
  - To find one \( x \in \mathbb{D} \) such that \( P(x) \) is false

\[ \exists x \in \mathbb{D}, \quad P(x) \]

- **True:**
  - You just need to find one \( x \) that satisfies \( P \)

- **False:**
  - \( P(x) \) is false for all \( x \) in \( \mathbb{D} \)
Negate quantifiers:

\[ \neg A \]

\[ \forall x \in D, \ p(x) \text{ is true} \quad \exists x \in D, \ p(x) \text{ is false} \]

\[ \exists x \in D, \ p(x) \text{ is true} \quad \forall x \in D, \ p(x) \text{ is false} \]

So, the equation is:

\[ 2x + 5 = 11 \]

Implicit in this sentence:

\[ x \in \mathbb{R} \]

"Solve" should be interpreted as show that there exists a real number \( x \) so that \( 2x + 5 = 11 \).