Let \( n \) be an integer. Show that if \( 2m^2 + n + 9 \) is odd then \( n \) is even.

\( A: \) \( 2m^2 + n + 9 \) is odd \( \rightarrow A: 2m^2 + n + 9 \) is even

\( B: \) \( n \) is even \( \rightarrow B: n \) is odd

\[ \text{Indirect proof: I assume } \neg B \text{ is true meaning } n \text{ is odd. There exists an integer } k \text{ such that } n = 2k + 1. \]

\[ 2m^2 + n + 9 = 2 \left( (2k+1)^2 + 2k+1 + 9 \right) \]
\[ = 2 \left( 4k^2 + 4k + 1 + 2k + 1 + 9 \right) \]
\[ = 2 \left( 4k^2 + 6k + 12 \right) \]
\[ = 2 \left( 4k^2 + 5k + 6 \right) \]

\( 4k^2 + 5k + 6 \) is an integer. Therefore \( 2m^2 + n + 9 \) is even meaning \( \neg A \) is true.
Direct proof?

I assume that \( A \) is true, meaning \( 2m^2 + n + 9 \) is odd.

There exists an integer \( k \) such that
\[
2m^2 + n + 9 = 2k + 1
\]

\[
m = 2k + 1 - 2m^2 - 9
\]
\[
= 2k - 2m^2 - 8
\]
\[
= 2(k - m^2 - 4)
\]

Since \( k - m^2 - 4 \) is an integer, \( n \) is even.

Therefore \( B \) is true. \( A \implies B \) is true.
\[ A \rightarrow B, \quad \neg B \rightarrow \neg A, \quad \neg A \rightarrow \neg B \]

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If John is a poet, then John is poor.

**Problem:**

Let \( a \) and \( b \) be two integers. Show that if \( a^2 (b^2 - 2b) \) is odd, then \( a \) is odd and \( b \) is odd.

A: \( a^2 (b^2 - 2b) \) is odd \(-A: a^2 (b^2 - 2b) \) is even

B: \( a \) is odd and \( b \) is odd \(-B: a \) is even or \( b \) is even.

\(-B\) seems easier to use than \( A \). I will use an indirect proof.
I want to show \( \neg B \Rightarrow \neg A \).

\( \neg B \): \( a \) is even or \( b \) is even.

\( \neg A \): \( a^2 (b^2 - 2b) \) is even.

**Assumption:** \( \neg B \) is true.

**Case 1:** \( a \) is even.

There exists an integer \( k \) such that \( a = 2k \).

\[ a^2 (b^2 - 2b) = 4k^2 (b^2 - 2b) \]

\[ = 2 \left[ 2k^2 (b^2 - 2b) \right] \]

\( 2k^2 (b^2 - 2b) \) is an integer, \( a^2 (b^2 - 2b) \) is even.

In Case 1, \( \neg A \) is true.

**Case 2:** \( b \) is even.

There exists an integer \( k \) such that \( b = 2k \).

\[ a^2 (b^2 - 2b) = a^2 (4k^2 - 4k) = 2 \left[ a^2 (2k^2 - 2k) \right] \]

\( a^2 (2k^2 - 2k) \) is an integer. Therefore \( a^2 (b^2 - 2b) \) is even.

In Case 2, \( \neg A \) is true.

In all cases covered by \( \neg B \), we have shown that \( \neg A \) is true. Therefore \( \neg B \Rightarrow \neg A \) is true and, by contrapositive, \( A \Rightarrow B \) is true.
Let \( a \) and \( b \) be two integers. Show that if either \( ab \) or \( a+b \) is odd, then either \( a \) or \( b \) is odd.

**A:** \( ab \) is odd \( a+b \) is odd \( \neg A: \) \( a \) is even and \( b \) is even

**B:** \( a \) is odd \( b \) is odd \( \neg B: \) \( a \) is even and \( b \) is even

**Indirect proof:**

Assumption: \( \neg B \) is true, meaning \( a \) is even and \( b \) is even.

There exists an integer \( k \) such that \( a = 2k \).

There exists an integer \( p \) such that \( b = 2p \).

i) We show \( ab \) is even

\[ ab = 2k \times 2p = 2(2kp) \]

As \( 2kp \) is an integer, \( ab \) is even.

ii) We show \( a+b \) is even

\[ a+b = 2k + 2p = 2(k+p) \]

\( k+p \) is an integer, therefore \( a+b \) is even.

We have shown that \( \neg A \) is true.

Therefore \( \neg B \to \neg A \) is true, and \( A \to B \) is true.
Let $a$ and $b$ be two integers. Use a direct proof to show that if $a^2 + b^2$ is even, then $a + b$ is even.

**A:** $a^2 + b^2$ is even

**B:** $a + b$ is even

**Direct proof:** I assume that $A$ is true, meaning $a^2 + b^2$ is even. There exists an integer $k$ such that $a^2 + b^2 = 2k$.

$$(a+b)^2 = a^2 + b^2 + 2ab$$

$$= 2k + 2ab$$

$$= 2(k + ab)$$

$k + ab$ is an integer. Therefore $(a + b)^2$ is even.

$(a + b)^2$ is even

If $m^2$ is even, then $m$ is even. Therefore $(a + b)$ is even.