Homework 1 - For 1/18/2023

Exercise 1 (10 points)
Fifty-six biscuits are to be fed to ten pets; each pet is either a cat or a dog. Each dog is to get six biscuits, and each cat is to get five. How many dogs and how many cats are there?

Exercise 2 (10 points)
A baseball and a baseball bat cost $1.10. The bat costs $1.0 more than the ball. How much does the ball cost?

Exercise 3 (10 points)
There are 100 cats living in a barn, all are either brown or orange. There is at least one orange cat. For every two cats, at least one is brown. How many orange cats and how many brown cats are there?

Exercise 4 (5 points each; total 20 points)
Prove the following identities for \( p, q, m, n, x, \) and \( y \) real numbers:

a) \( 8(p - q) + 4(p + q) = 2(p + 3q) + 10(p - q) \)

b) \( x(m - n) + y(n + m) = m(x + y) + n(y - x) \)

c) \( (x + 3)(x + 8) - (x - 6)(x - 4) = 21x \)

d) \( m^8 - 1 = (m^2 - 1)(m^2 + 1)(m^4 + 1) \)
Exercise 5 *(10 points)*

Let us play a logical game. You find yourself in front of three rooms whose doors are closed. One of these rooms contains a Lady, another a Tiger and the third room is empty. There is one sign on each door; you are told that the sign on the door of the room containing the Lady is true, the sign on the door of the room with the Tiger is false, and the sign on the door of the empty room could be either true or false. Here are the signs:

<table>
<thead>
<tr>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Room III is empty</td>
<td>The Tiger is in room I</td>
<td>This room is empty</td>
</tr>
</tbody>
</table>

Figure 1: The three rooms and their signs

Can you find what is behind each door?

Exercise 6 *(10 points)*

You are on an island inhabited by three types of people: knights (always make true statements), knaves (always make false statements) and spies (sometimes make true statements and sometimes make false statements). You come across 3 people Adam, Ben and Carl. You know that one is a knight, one is a knave and one is a spy. They say the following:

- Adam: “I am not a knight”
- Ben: “I am not a knave”
- Carl: “I am not a spy”

Determine which person is what or whether you do not have enough information.