Homework 4 - For 2/8/2023

Exercise 1 (10 points)

Let us play a logical game. You find yourself in front of three rooms whose doors are closed. One of these rooms contains a Lady, another a Tiger and the third room in empty. There is one sign on each door; you are told that the sign on the door of the room containing the Lady is true, the sign on the door of the room with the Tiger is false, and the sign on the door of the empty room could be either true or false. Here are the signs:

<table>
<thead>
<tr>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Room III is empty</td>
<td>The Tiger is in room I</td>
<td>This room is empty</td>
</tr>
</tbody>
</table>

Figure 1: The three rooms and their signs

Can you find out what is behind each door?

Exercise 2 (10 points)

We are on the island of knights and knaves. You meet two residents, John and Bill who make the following statements:

John says: “If Bill is a knave, then I am a knight”
Bill says: “We are different”

Can you find what are John and Bill?

Exercise 3 (10 points)

A very special island is inhabited only by knights and knaves. Knights always tell the truth, and knaves always lie. You meet three inhabitants: Alex, John and Sally. Alex says, “At least one of
the following is true: that Sally is a knave or that I am a knight.” John says, ”Alex could claim that I am a knave.” Sally claims, “Neither Alex nor John are knights.” Can you find who is a knight and who is a knave?

Exercise 4 (10 points)

Design a circuit that implements majority voting for three individuals (i.e. the output of the circuit is 1 if two at least of the inputs are 1, and 0 otherwise). (Hint: consider the Boolean expression \((A + B) \cdot C + (A + C) \cdot B\).

Exercise 5 (2 parts, each 10 points; Total 20 points)

a) Assume that you only have NAND gate at your disposal. Show that the following gate is equivalent to the OR gate.

b) Draw a diagram whereby multiple NAND gates are connected together to form an AND gate. (Hint: the NOT gate can be formed using:)

Exercise 6 (10 points)

Show that the proposition \(P = [(r \lor p) \rightarrow (r \lor q)] \leftrightarrow [r \lor (p \rightarrow q)]\) is a tautology.

Exercise 7 (10 points)

Show that the following is a tautology: \(((p \lor q) \land (p \rightarrow r) \land (q \rightarrow r)) \rightarrow r\).