Homework 4 - For 2/2/2022

Exercise 1 (10 points)

Let us play a logical game. You find yourself in front of three rooms whose doors are closed. One of these rooms contains a Lady, another a Tiger and the third room is empty. There is one sign on each door; you are told that the sign on the door of the room containing the Lady is true, the sign on the door of the room with the Tiger is false, and the sign on the door of the empty room could be either true or false. Here are the signs:

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<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>I</td>
<td>Room III is empty</td>
<td>II</td>
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</tbody>
</table>

Figure 1: The three rooms and their signs

Can you find out what is behind each door?

Exercise 2 (10 points each; total 20 points)

Construct a truth table for each of these compound propositions:

a) \( A = (\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r) \)

b) \( (p \oplus q) \land (p \oplus \neg q) \)

Exercise 3 (10 points)

Show that the proposition \( P = [(r \lor p) \rightarrow (r \lor q)] \leftrightarrow [r \lor (p \rightarrow q)] \) is a tautology.

Exercise 4 (10 points)

Show that the following is a tautology: \( [(p \lor q) \land (p \rightarrow r) \land (q \rightarrow r)] \rightarrow r \).
Exercise 5 (10 points)

Use either a truth table or logical equivalence to show that \((p \rightarrow q) \land (p \rightarrow r) \iff p \rightarrow (q \land r)\)

Exercise 6 (10 points)

We are on the island of knights and knaves. You meet two residents, John and Bill who make the following statements:

John says: “If Bill is a knave, then I am a knight”
Bill says: “We are different”

Can you find what are John and Bill?