Homework 5 - For 2/15/2023

Exercise 1: (10 points each; total 20 points)

Let $p$, $q$, and $r$ be three propositions. Using truth tables or logical equivalences, indicate which (if any) of the propositions below are tautologies, contradictions, or neither.

a) $A = (p \land q) \lor r \lor (\neg q \land \neg r) \lor (\neg p \land \neg r)$

b) $[p \lor (q \rightarrow r)] \rightarrow (p \lor q \lor r)$

Exercise 2 (10 points)

Prove that if $n$ is a positive integer, then $n$ is even if and only if $7n + 4$ is even.

Exercise 3 (10 points)

Let $a$ and $b$ be two positive integers. Prove that if $n = ab$, then $a \leq \sqrt{n}$ or $b \leq \sqrt{n}$

Exercise 4 (10 points)

Let $m$ and $n$ be two integers. Show that if $m > 0$ and $n \leq -2$, then $m^2 + mn + n^2 \geq 0$

Exercise 5 (10 points each; total 30 points)

Let $a$ and $b$ be two integers. Show that if $a^2 + b^2$ is even, then $a + b$ is even:

a) Using an indirect proof (proof by contrapositive)

b) Using a proof by contradiction

c) Using a direct proof
Exercise 6 (10 points)

The Fair Maiden Rowena wishes to wed. And her father, the Evil King Berman, has devised a way to drive off suitors. He has a little quiz for them, and here it is. It’s very simple:

Three boxes sit on a table. The first is made of gold, the second is made of silver, and the third is made of lead. Inside one of these boxes is a picture of the fair Rowena. It is the job of the White Knight to figure out, without opening them, which one has her picture.

Now, to assist him in this endeavor there is an inscription on each of the boxes. The gold box says, “Rowena’s picture is in this box.” The silver box says, “The picture is not in this box.” The lead box says, “The picture is not in the gold box.” Only one of the statements is true. Which box holds the picture?