Homework 6 - For 2/23/2022

Exercise 1: 10 points

Using induction, show that $\forall n \in \mathbb{N}, \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.

Exercise 2: 10 points

Using induction, show that $\forall n \in \mathbb{N}, \sum_{i=1}^{n} i(i+1)(i+2) = \frac{n(n+1)(n+2)(n+3)}{4}$.

Exercise 3: 10 points

Show that $\forall n \in \mathbb{N}, n > 1, \sum_{i=1}^{n} \frac{1}{i^2} < 2 - \frac{1}{n}$.

Exercise 4: 10 points

Use a proof by induction to show that $\forall n \in \mathbb{N}, n > 3, n^2 - 7n + 12 \geq 0$.

Exercise 5: 10 points

Find the flaw with the following proof that : $P(n) : a^n = 1$ for all non negative integer $n$, whenever $a$ is a non zero real number:

- **Basis step**: $P(0)$ is true: $a^0 = 1$ is true, by definition of $a^0$.
- **Strong Inductive step**: assume that $a^j = 1$ for all non negative integers $j$ with $j \leq k$. Then note that:

$$a^{k+1} = \frac{a^k a^k}{a^{k-1}} = \frac{1 \times 1}{1} = 1$$

Therefore $P(k+1)$ is true.
The principle of proof by strong mathematical induction allows us to conclude that $P(n)$ is true for all $n \geq 0$.

**Exercise 6: 10 points**

Show that $\forall n \in \mathbb{N} f_1^2 + f_2^2 + \ldots + f_n^2 = f_n f_{n+1}$ where $f_n$ are the Fibonacci numbers.