Exercise 1

Let $p$ and $q$ be two propositions. The proposition $p \text{ NOR } q$ is true when both $p$ and $q$ are false, and it is false otherwise. It is denoted $p \downarrow q$

a) Write down the truth table for $p \downarrow q$

b) Show that $p \downarrow q$ is logically equivalent to $\neg(p \lor q)$

c) Find a compound proposition logically equivalent to $p \rightarrow q$ using only the logical operator $\downarrow$

Exercise 2

Let $P(x)$ be the statement “$x=x^2$”. If the domain consists of the integers, what are the truth values of the following statements:

a) $P(0)$
b) $P(1)$
c) $P(2)$
d) $P(-1)$
e) $\exists x \ P(x)$ f) $\forall x \ P(x)$

Exercise 3

Express each of these statements using quantifiers. Then form the negation of the statement so that no negation is to the left of a quantifier. Next, express the negation in simple English. (Do not simply use the phrase “It is not the case that.”)

a) All dogs have fleas.
b) There is a horse that can add.
c) Every koala can climb.
d) No monkey can speak French.
e) There exists a pig that can swim and catch fish.

Exercise 4

a) Let $a$ and $b$ be two real numbers. Prove that if $n = ab$, then $a \leq \sqrt{n}$ or $b \leq \sqrt{n}$

b) Prove or disprove that there is a rational number $x$ and an irrational number $y$ such that $x^y$ is irrational.

c) Prove that $\sqrt[3]{2}$ is irrational
d) There exists no integer $a$ and $b$ such that $21a + 30b = 1$