Exercise 0

Additional problems on proofs:
   a) Let \( x \) and \( y \) be two integers. Show that if \( 2x+5y=14 \) and \( y \neq 2 \), then \( x \neq 2 \).
   b) Let \( x \) and \( y \) be two integers. Show that if \( x^2+y^2 \) is odd, then \( x+y \) is odd.

Exercise 1

Determine whether each of these functions is a bijection from \( \mathbb{R} \) to \( \mathbb{R} \):
   a) \( f(x) = 2x+4 \)
   b) \( f(x) = x^2+1 \)
   c) \( f(x) = \frac{x+1}{x+2} \)
   d) \( f(x) = \frac{x^2+1}{x^2+2} \)

Exercise 2

Let \( S = \{-1,0,2,4,7\} \). Find \( f(S) \) if:
   a) \( f(x) = 1 \)
   b) \( f(x) = 2x+1 \)
   c) \( f(x) = \left\lfloor \frac{x}{5} \right\rfloor \)
   d) \( f(x) = \left\lfloor \frac{x^2+1}{3} \right\rfloor \)

Exercise 3

Let \( S \) be a subset of a universe \( U \). The characteristic function \( f_S \) of \( S \) is the function from \( U \) to the set \( \{0,1\} \) such that \( f_S(x) = 1 \) if \( x \) belongs to \( S \) and \( f_S(x) = 0 \) if \( x \) does not belong to \( S \). Let \( A \) and \( B \) be two sets. Show that for all \( x \) in \( U \),
   a) \( f_{A\cap B}(x) = f_A(x) \cdot f_B(x) \)
   b) \( f_{A\cup B}(x) = f_A(x) + f_B(x) - f_A(x) \cdot f_B(x) \)

Exercise 4

Let \( n \) be an integer. Show that \( \left\lfloor \frac{n}{2} \right\rfloor \left\lfloor \frac{n}{2} \right\rfloor = \left\lfloor \frac{n^2}{4} \right\rfloor \).