Exercise 1: proofs

a) Let $x$ and $y$ be two integers. Show that if $2x + 5y = 14$ and $y ≠ 2$, then $x ≠ 2$

b) Let $x$ and $y$ be two integers. Show that if $x^2 + y^2$ is odd, then $x + y$ is odd

Exercise 2: floor and ceiling

a) Let $x$ be a real number. Show that

$$\left\lfloor \frac{x}{2} \right\rfloor = \left\lfloor \frac{x}{4} \right\rfloor$$

b) Show that if $n$ is an odd integer, then

$$\left\lfloor \frac{n^2}{4} \right\rfloor = \frac{n^2 + 1}{3}$$

Exercise 3: Growth of functions

a) Show that if a function $f(x)$ from $\mathbb{R}$ to $\mathbb{R}$ is $O(x)$, then $f(x)$ is $O(x^2)$.

b) Show that the function $f(n) = n \log(n^2 + 1) + \frac{\log(n)}{n^2 + 1}$ is $O(n \log(n))$

Exercise 4: Algorithms

Describe an algorithm that interchanges the values of the variables $x$ and $y$, using only assignments.