New Problems

Induction

November 24, 2017

**Induction**

**Exercise a**
Show that

\[ \sum_{k=1}^{n} k2^k = (n-1)2^{n+1} + 2 \]

for all positive integer \( n \geq 1 \).

**Exercise b**
Show that \( 3^n < n! \) for all \( n > 6 \).

**Exercise c**
Prove that if \( n \geq 1 \), then 133 divides \( 11^{n+1} + 12^{2n-1} \).

**Exercise d**
Suppose the post office sold only five-cent stamps and seven-cent stamps. Show that any amount of postage strictly greater than 23 can be made with just those two kinds of stamps.

**Fibonacci**

**Exercise a**
Let \( f_n \) be the Fibonacci numbers. Show that

\[ f_0f_1 + f_1f_2 + f_2f_3 + \ldots + f_{2n-1}f_{2n} = f_{2n}^2 \]