ECS 20: Discrete Mathematics
Midterm
November 20, 2017

Notes:
1) Midterm is open book, open notes. No computers though…
2) You have 45 minutes, no more: We will strictly enforce this.
3) You can answer directly on these sheets (preferred), or on loose paper.
4) Please write your name at the top right of at least the first page that you turn in!
5) Please, check your work!

Part I: sets (1 question, 10 points)

Let $A$, $B$, and $C$ be three sets in a domain $D$. Show that $A \cup (\overline{A} \cap B) \cup (\overline{A} \cap \overline{B} \cap C) = A \cup B \cup C$
Part II: functions (2 questions; each 10 points; total 20 points)

1) Let \(x\) and \(y\) be two real numbers such that \(x \notin \mathbb{Z}\) and \(y \notin \mathbb{Z}\) but \(x + y \in \mathbb{Z}\). Show that 
\[
[x] + [y] = x + y - 1.
\]

2) Let \(n\) be a natural number and let \(x\) be a real number. Show that 
\[
\left\lfloor \frac{nx}{n} \right\rfloor = [x]
\]
Part III: Number theory (3 questions; each 10 points; total 30 points)

1) Let $a$, $b$, and $c$ be three natural numbers such that $\gcd(a,b)=1$ and $a/(bc)$ and $b/(ac)$ and $c/(ab)$.
   a) Show that $a/c$ and $b/c$ (4 points)

   b) Show that $c = ab$ (6 points)
2) Show that the remainder of the long division of the square of an odd integer by 8 is always 1.

3) What is the remainder of the division of $14^{3141}$ by 17?
Part IV: extra credit (3 points)

a) (1 point) Let $n$ be a natural number. Show that $5^n$ is odd.

b) (2 points) Using the result from a), show that $\log_{10}(2)$ is irrational.