Homework 3: due 10/19/2017

ECS 20 (Fall 2017)

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Exercise 1

Show that this implication is a tautology, using a truth table:

$$(p \lor q) \land (\neg p \lor r) \rightarrow (q \lor r)$$

Exercise 2

Show that $(p \land q) \rightarrow r$ and $(p \rightarrow r) \land (q \rightarrow r)$ are not logically equivalent.

Exercise 3

Determine the truth values of the following statements; justify your answers:

a) $\forall n \in \mathbb{N}, (n + 2) > n$

b) $\exists n \in \mathbb{N}, 2n = 3n$

c) $\forall n \in \mathbb{Z}, 3n \leq 4n$

d) $\exists x \in \mathbb{R}, x^4 < x^2$

Exercise 4

Show that the following statements are true.

a) Let $x$ be a real number. Prove that if $x^3$ is irrational, then $x$ is irrational.

b) Let $x$ be a positive real number. Prove that if $x$ is irrational, then $\sqrt{x}$ is irrational.

c) Prove or disprove that if $a$ and $b$ are two rational numbers, then $a^b$ is also a rational number.

d) Let $n$ be a natural number. Show that $n$ is even if and only if $3n + 8$ is even.

e) Prove that either $4 \times 10^{769} + 22$ or $4 \times 10^{769} + 23$ is not a perfect square. Is your prove constructive, or non-constructive?

Note: for question e), a natural number $n$ is a perfect square if there exists a natural number $q$ such that $n = q^2$. For example, 4, 9, 16, 25, .... are all perfect squares while 2, 3, 5, 6,... are not.
Exercise 5

Let $n$ be a natural number and let $a_1, a_2, \ldots, a_n$ be a set of $n$ real numbers. Prove that at least one of these numbers is greater than, or equal to the average of these numbers. What kind of proof did you use?

Exercise 6

Let $n$ be an integer. Show that if $n^3 + 7$ is even, the $n$ is odd, using:

a) a proof by contraposition

b) a proof by contradiction

Extra Credit

Use Exercise 5 to show that if the first 10 strictly positive integers are placed around a circle, in any order, then there exist three integers in consecutive locations around the circle that have a sum greater than or equal to 17.