Exercise 1: total 20 points (5 points for each of a to d)

a) Show that $4x - 10$ is $\Theta(x)$.

b) Show that $4x^2 + 2x - 2$ is $\Theta(x^2)$.

c) Show that $\lfloor x + \frac{2}{3} \rfloor$ is $\Theta(x)$.

d) Show that $\log_5(x)$ is $\Theta(\log_3(x))$.

Exercise 2: 10 points

Show that $x^2$ is $O(x^4)$ but that $x^4$ is not $O(x^2)$.

Exercise 3: 10 points

Let $a$, and $b$ be two strictly positive integers and let $x$ be a real number. Show that:

$$\left\lfloor \frac{\lfloor x \rfloor}{b} \right\rfloor = \lfloor \frac{x}{ab} \rfloor$$

Exercise 4: 10 points

Let $x$ be a positive real number. Solve $\lfloor x \lfloor x \rfloor \rfloor = 5$.

Exercise 5: 10 points

Let $n$ be a natural number. Show that if $n$ is a perfect square, then $2n$ is not a perfect square.

(Reminder: a natural number $a$ is a perfect square if there exists a natural number $k$ such that $n = k^2$.)

Extra Credit: 5 points

Find all functions $f : \mathbb{R} \to \mathbb{R}$ that satisfy:

$$\forall (x, y) \in \mathbb{R}^2, f(x)f(y) + f(x + y) = xy$$