ECS20
Homework 6

Exercise 1

Prove or disprove each of these statements about the floor and ceiling functions.

a) \(\lfloor x \rfloor = x \) for all real numbers \(x\).

b) \(\lfloor xy \rfloor = \lfloor x \rfloor \lfloor y \rfloor\) for all real numbers \(x\) and \(y\).

c) \(\lfloor \sqrt{x} \rfloor = \lfloor \sqrt{x} \rfloor\) for all positive real numbers \(x\).

Exercise 2

Show that \(x^3\) is \(O(x^4)\) but that \(x^4\) is not \(O(x^3)\).

Exercise 3

a) Show that \(2x-9\) is \(\Theta(x)\).

b) Show that \(3x^2 + x - 5\) is \(\Theta(x^2)\).

c) Show that \(\lfloor x + \frac{2}{3} \rfloor\) is \(\Theta(x)\).

d) Show that \(\log_{10}(x)\) is \(\Theta(\log_2(x))\).

Exercise 4

Let \(a\) and \(b\) be two integers. Use a proof by contradiction to show that if \(a^2 - b^2 + 2ab\) is odd then \(a-b\) is odd.

Exercise 5

Use a proof by contradiction to show that:

There exists a strictly positive real number \(r\) such that for all real number \(x\), if \(x - \lfloor x \rfloor < r\) then \(\lfloor 3x \rfloor = 3x\).

Extra credit:

We call a positive integer perfect if it equals the sum of its positive divisors other than itself.

a) Show that 6 and 28 are perfect

b) Show that \(2^{p-1}(2^p - 1)\) is a perfect number when \(2^p - 1\) is prime.