Exercise 1:

What are the quotient and remainder when:

a) -2002 is divided by 89?
b) 0 is divided by 19?
c) 1,234,567 is divided by 101?
d) -100 is divided by 103?

Exercise 2:

a) Let a be a positive integer. Show that \( \text{gcd}(a,a-1) = 1 \).
b) Use the result of part a) to solve the Diophantine equation \( a + 2b = 2ab \) where (a,b) are positive integers.

Exercise 3:

Let \( a, b, \) and \( c \) be three integers. Show that the equation \( ax + by = c \) has at least one solution \((x_1,y_1)\) if and only if \( \text{gcd}(a,b) / c \).

Exercise 4:

Let \( a, b \) and \( n \) be three positive integers with \( \text{gcd}(a,n) = 1 \) and \( \text{gcd}(b,n) = 1 \). Show that \( \text{gcd}(ab,n) = 1 \).

Exercise 5:

Prove that there are no solutions in integers \( x \) and \( y \) to the equation \( 3x^2 + 5y^2 = 19 \). (*Hint:* consider this equation modulo 5)

Exercise 6:

Show that if \( n > 3 \) then \( n, 2n+1 \) and \( 4n+1 \) cannot all be prime (*Hint:* consider the division of \( n \) by 3)

Exercise 7:

Prove or disprove that there are three consecutive odd positive integers that are primes, that is, odd primes of the form \( p, p+2, p+4 \).

Exercise 8:
Prove that if $n$ is a positive integer such that the sum of its divisors is $n+1$, then $n$ is prime.

**Extra credit:**

Let $a$ and $b$ be two strictly positive integers. Solve $gcd(a,b) + lcm(a,b) = b + 9$